B.A./B.Sc. 5th Semester (Honours) Examination, 2022 (CBCS)

Subject : Mathematics

Course : BMH5CCXII

(Mechanics-I)

Time: 3 Hours **Full Marks: 60**

The figures in the margin indicate full marks.

Candidates are required to give their answers in their own words as far as practicable.

Notation and symbols bear usual meaning.

1. Answer *any ten* questions from the following: 2×10=20

(a) Define the angle of friction and cone of friction.

(b) Define astatic centre for a system of coplanar forces.

(c) When an equilibrium is said to be unstable equilibrium?

(d) Write down the conditions of equilibrium of a system of non-coplanar forces.

(e) Prove that a central orbit is a plane curve.

(f) State D'Alembart's principle.

(g) Find the resultant of two simple harmonic motions having slightly different periods.

(h) State Kepler's laws of planetary motion.

(i) Define a singular point of a central orbit. When will it be a cusp?

(j) State the principle of conservation of linear momentum for a system of conservative forces.

(k) Find the moment of inertia of an elliptic lamina about an axis through the centre and perpendicular to the lamina.

(l) Two particles move under their mutual gravitational attraction. Write down their equations of motion in vector form.

(m) Explain the concept of 'Momental ellipsoid'.

(n) Give an example of a non-conservative force. Justify your answer.

(o) Write down the expressions of potential energy and kinetic energy of a simple pendulum of length l oscillating in a uniform gravitational field.

2. Answer *any four* questions from the following: 5×4=20

(a) Prove that in a catenary of uniform strength $s = a\log(\sec\Psi + \tan\Psi)$, $\rho = a\sec\Psi$, $\rho = a\cosh\left(\frac{s}{a}\right)$. Hence show that the mass per unit length at any point varies as the corresponding radius of curvature. 5

(b) State the laws of limiting friction. A hemisphere shell rests on a rough plane, whose angle of frictions is λ, show that the inclination of the plane base of the rim to the horizontal cannot be greater than $\sin^{-1}(2\sin\lambda)$. 2+3

(c) Show that the differential equation of the central orbit in polar co-ordinates is $\frac{h^2}{p^3}\frac{dp}{dr} = F$, where the symbols have their usual meaning. 5

(d) A system of n particles is moving under external forces and their mutual action and reactions. Write down the equation of motion of the system of particles and obtain the equation of motion of the centre of mass. 1+4

(e) A solid frustum of a paraboloid of revolution, of height h and latus rectum $4a$, rests with its vertex on the vertex of a paraboloid of revolution, whose latus rectum is $4b$; show that the equilibrium is stable if $h < \frac{3ab}{(a+b)}$. 5

(f) A particle moves in a path so that its acceleration is always directed to a fixed point and is equal to $\mu/(\text{distance})^2$; so that its path is a conic section and distinguish between the three cases that arise. 3+2

3. Answer *any two* questions from the following: 10×2=20

(a) (i) State the energy test of stability.

(ii) Establish the energy test of stability of equilibrium of a body with one degree of freedom.

(iii) Show that the momental ellipsoid at the centre of an ellipsoid is given by the equation $(b^2 + c^2)x^2 + (c^2 + a^2)y^2 + (a^2 + b^2)z^2 = \text{contant}$. 2+4+4

(b) (i) A particle falls under gravity (supposed constant) in a resisting medium whose resistance varies as the square of the velocity. If the particle starts from rest, find the distance through which it has fallen in time t.

(ii) A particle moves with a central acceleration $\frac{\mu}{r^2} - \frac{\lambda}{r^3}$, where r is the distance from the centre and λ, μ are constants. Show that the apsidal angle is $\pi + \sqrt{1 + \frac{\lambda}{h^2}}$, where $\frac{h}{2}$ is the constant areal velocity. 5+5

(c) (i) A force P acts along the axis of x and another force nP along a generator of the cylinder $x^2 + y^2 = a^2$, show that the central axis lies on the cylinder $n^2(nx - z)^2 + (1 + n^2)^2y^2 = a^2n^4$.

(ii) A square frame work formed of uniform heavy rods of equal weights W joined together, is hung up by one corner. A weight W is suspended from each of the three lower corner and the shape of the square is preserved by a light rod along the horizontal diagonal, prove that its thrust is $4W$. 5+5

(d) (i) Show that the moment of momentum of a rigid body moving in two dimensions is given by $Mvp + Mk^2\dot{\theta}$, where symbols have their usual meaning.

(ii) A homogeneous sphere, of radius a, rotating with angular velocity ω about a horizontal diameter is placed on a plane table whose coefficient of friction is μ. Show that there will be slipping at the point of contact for a time $2\omega a/7\mu g$ and then the sphere will roll with angular velocity $2\omega/7$. 5+5
